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THERMODYNAMIC CONSTRAINTS ON VISCOELASTIC MODELS FOR WAVE ABSORBING MATERIALS

S. GOROG*
Faculté des Sciences appliquées, Blvd Université, Sherbrooke, Québec, Canada J1K 2R1

M. P. LUONG
CNRS-Laboratoire de Mécanique des Solides, Ecole Polytechnique, 91128 Palaiseau Cedex, France

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Abstract—The present paper analyses models describing wave absorbing materials from a thermodynamic point of view. This study deals with harmonic plane wave propagation through a viscoelastic homogeneous medium at the macroscopic scale. The dynamic behaviour of the studied medium is modelled using two distinct complex functions related to the bulk and to the density, respectively. It is shown that the complex density function results from viscous body forces. This paper aims to discuss the thermodynamic constraints on these acoustic models for a general one-dimensional (1D)-harmonic plane wave system. The dynamic intrinsic dissipation of the studied viscoelastic medium is defined and evaluated. Using the second law of thermodynamics, conditions are found to define consistent functions modelling the dissipative effects. The classical rigid open porous model used to describe many sound absorbing media is taken as an example for such a harmonic viscoelastic model. It demonstrates that the main dissipative phenomenon can be described using a complex density function. © 1998 Elsevier Science Ltd. All rights reserved.

1. INTRODUCTION

For many industrial problems, the computational methods elaborated to predict acoustic and vibratory responses of complex structures (aircraft panels, building walls, vehicle interiors...) use constitutive laws which have to realistically model the dynamic behaviour of concerned materials. These models are usually developed in the framework of the linear viscoelasticity and, therefore, elaborated in the frequency domain. In order to take into account particular damping effects, a complex (in the mathematical sense) mass quantity is readily introduced into the numerical procedure.

This paper discusses the thermodynamic constraints on such viscoelastic models describing the dynamic behaviour of wave absorbing materials at the macroscopic scale. Many studies on thermodynamics of viscoelastic laws can be found, but few have been developed on the harmonic aspect of the thermodynamic analysis. Caviglia and Morro (1991, 1993) show that inhomogeneities of the solid suggest to introduce complex mass density. However, the restriction of thermodynamic character is applied on the viscoelastic tensor and the main discussion is to analyse the decay of the wave. Dvostam (1995) proposed to take into account a damping effect, introducing adding frequency complex anelastic terms to the material modulus matrix of Hooke's generalised law. For that purpose, harmonic calculation of the entropy production is given.

Models used to analyse the wave propagation through sound absorbing media, such as polyurethane foams or glass wools, are directly established in the frequency domain as presented in the well-known works of Zwikker and Kosten (1949), Attenborough (1982) and Allard (1993). For these acoustic studies, harmonic consideration leads to define two complex descriptive functions related to the viscous bulk rigidity and to a mass effect. These models are then assumed to be well defined from the point of view of mechanics of continua.

* Author to whom correspondence should be addressed. E-mail: sylvie.gorog@gme.usherb.ca.

This paper attempts to analyse such acoustic models from the thermodynamic point of view. It is assumed, first, that the studied material is described as a homogeneous viscoelastic medium at the scale of the continuum and second, that its dynamic behaviour is given by a resulting harmonic response. Hence, the present work deals with the modelling of the complex descriptive functions and aims to give the thermodynamic restrictions needed to define a consistent viscoelastic harmonic model by applying the second law of the thermodynamics of irreversible processes (Maugin, 1992). It is chosen, therefore, to use the classical tools of the continuum mechanics in the frequency domain with the assumption of small motions.

As a starting point, Section 2 presents two frequency dependent functions modelling the viscoelastic medium excited by a harmonic source of angular frequency ω. The first complex function represents an effective density while the second function describes the viscous bulk effects. The time-viscoelastic constitutive law gives a linear frequency relationship between stress and strain defining a complex rigidity function by use of the generalised Hooke's tensor (Hunter, 1960).

By taking into account viscous body forces in the equation of motion, it is shown that a complex density function can be defined in the frequency domain. It is assumed that both functions result from a phenomenological approach at the scale of the microstructure. For instance, in the case of the sound absorbing materials, the microstructure is modelled as an open porous network saturated by the ambient air. Therefore, the density function takes into account the internal viscous fluid flow. Hence, at the scale of the continuum, the complex density function is presented here as the result of a second constitutive law related to body forces. It is also shown that the pair of the mechanical descriptive functions can be replaced by the two classical acoustic parameters, the characteristic impedance and the wave number (Morse and Ingard, 1968). For a three-dimensional (3D)-description of solids, the present paper proposes to define the characteristic impedance using an impedance vector.

The aim of the work presented is to evaluate the dissipated energy of a viscoelastic wall submitted to an acoustical or vibratory source. For the sake of simplicity, a one-dimensional propagation system (Section 3) is proposed to develop the thermodynamic analysis. It concerns an acoustic wall of finite thickness, excited on both sides by normal incident harmonic plane waves. It is assumed that both media on both sides of the studied wall are viscoelastic and well known. Moreover, it has been decided to describe these external media by the proposed model using two complex functions. It is equivalent to study the dynamic behaviour of a layer in a viscoelastic stratified structure during the wave propagation.

To apply the second law of thermodynamics, it is necessary to define the energy dissipated through the studied medium. The intrinsic dissipation is deduced from the energy balance (Section 4) related to the propagation system. Using the thermodynamic definitions, an internal frequency dependent dissipation is defined in relation to the theorem of the virtual work rate (Maugin, 1992). This energy balance is applied on the studied continuum modelling the wall.

Section 5 presents the calculation of the intrinsic dissipation for the harmonic system under consideration. It is shown that the two descriptive functions representing different viscous effects permit to highlight two kinds of dynamic dissipations.

Applying the second law of thermodynamics, the positivity of the calculated dissipation is discussed in Section 6. It gives thermodynamic restrictions applied to the chosen model. For all frequencies, the main results are firstly, that real parts of both descriptive functions have to be positive and, secondly, that the imaginary part of the density function must be negative whereas the imaginary part of the bulk function has to be positive.

As an illustrative example, it is proposed to apply the present formulation using a model established by Allard (1993) which describes an open porous medium with a rigid skeleton (Section 7). Such a model is consistent with all thermodynamic conditions. This example demonstrates that the main dynamic dissipative effect can be described using the density function.

Finally, Section 8 is devoted to the conclusions.

2. VISCOELASTIC MODEL AND HARMONIC MOTION

Let $u(x, t)$ be the displacement vector of the particle located at the point x of the homogeneous viscoelastic medium under consideration. The particle acceleration is denoted by $a(x, t) = \ddot{u}(x, t)$. The classical equation of motion is given by:

$$\operatorname{div} \sigma(x, t) + f(x, t) = \rho a(x, t), \tag{1}$$

where $\sigma(x, t)$ represents the internal stress tensor, ρ the density of the medium, and $f(x, t)$ the body force vector.

It is assumed that the body forces result from the description of a physical phenomenon such as the conduction fluid flow for an open porous medium (Coussy, 1995). It follows that the vector $f(x, t)$ can be dependent on velocity and/or on acceleration of the particles, such that $f(x, t) = f(\dot{u}(x, t), \ddot{u}(x, t))$. This description can be understood as a second constitutive law related to kinetic or inertial internal effects. Hence, using for instance a filter function $\lambda(t)$ modelling an internal motion resulting from an external influence, the body force can be expressed as follows in the time domain:

$$f(x, t) = \lambda(t) \odot \dot{u}(x, t) \tag{2}$$

where $\odot$ designates the convolution operator.

Applying the harmonic time convention such that $u(x, t) = u^*(x) \exp[j\omega t]$ with $j^2 = -1$, and, using the Fourier transform, the dynamic behaviour is described in the frequency domain. Asterisks denote complex quantities. Including the harmonic body forces in the right hand side of eqn (1), the frequency equation of motion takes the following form:

$$\operatorname{div} \sigma^*(x) = \rho^*(\omega) a^*(x). \tag{3}$$

Function $\rho^*(\omega)$ represents a complex density function which results from the spectral transform of the body force. For instance, using eqn (2), the density function becomes:

$$\rho^*(\omega) = \rho - j\lambda^*(\omega)/\omega.$$

At the chosen descriptive scale of the continuum, this complex density is interpreted as a model of internal viscous mass effects for sound absorbing media (Allard, 1993).

In the frequency domain, the viscoelastic constitutive law gives a linear Hooke's relationship between the complex strain tensor $\varepsilon^*(x) = \frac{1}{2}(\operatorname{grad} u + {}^t\operatorname{grad} u)$ and the complex stress tensor $\sigma^*(x)$ (Hunter, 1960):

$$\sigma^*(x) = G^*(\omega) : \varepsilon^*(x), \tag{4}$$

where the tensor of the fourth order $G^*(\omega)$ is the generalised Hooke's tensor taking into account the viscous damping related to the bulk rigidity of the medium.

As a result, a class of harmonic behaviour models can be defined using the following two independent complex functions:

$$G^*(\omega) = G^{\mathrm{r}}(\omega) + jG^{\mathrm{i}}(\omega), \tag{5}$$

$$\rho^*(\omega) = \rho^{\mathrm{r}}(\omega) + j\rho^{\mathrm{i}}(\omega). \tag{6}$$

These functions express two different kinds of viscoelastic phenomena, the delayed mechanical response and the internal inertia effects, respectively. For elastic behaviours, the imaginary parts of these functions disappear.

The two classical parameters used to describe the harmonic wave propagation through a medium are, firstly, the characteristic impedance and secondly, the wave number (Caviglia and Morro, 1992). Using the law eqn (4), the equation of motion eqn (3) becomes:

$$G^*(\omega) \vdots \nabla\nabla u^*(x) = -\omega^2 \rho^*(\omega) u^*(x) \tag{7}$$

whose the general solution for an infinite plane wave propagation is:

$$u^*(x) = A\mathbf{p}\exp[-j\mathbf{k}^* \cdot x] \tag{8}$$

where A is the wave amplitude and $\mathbf{p}$ the normalised polarisation vector of the propagating wave. The wave number is a complex vector $\mathbf{k}^*$ whose direction is given by the normalised vector $\boldsymbol{\kappa}$ and whose real and imaginary parts characterise celerity and decay of the wave, respectively.

For an infinite 3D configuration, we propose to define the characteristic impedance as the impedance vector $\mathbf{z}$ in the time domain such that:

$$\mathbf{z} = \frac{-\sigma \cdot \mathbf{p}}{\dot{u} \cdot \mathbf{p}} \tag{9}$$

For harmonic motions, this impedance vector is complex and becomes (Gorog, 1995):

$$\omega \mathbf{z}^* = \tfrac{1}{2}\|\mathbf{k}^*\| G^* :(\mathbf{p} \otimes \boldsymbol{\kappa} + \boldsymbol{\kappa} \otimes \mathbf{p}) \cdot \mathbf{p}. \tag{10}$$

where $\otimes$ denotes the tensorial product.

As a result, it is equivalent to describe the viscoelastic medium either with the pair of functions ($G^*(\omega)$, $\rho^*(\omega)$) (mechanical description) or with the pair of functions ($\mathbf{k}^*(\omega)$, $\mathbf{z}^*(\omega)$) (acoustic description). According to harmonic motions, let us use the asterisks only for the wave number and the quantities directly defined by eqn (5), (6) and (10). Note that the classical viscoelastic models lead to define complex wave vector and impedance without complex density function. To study the thermodynamic constraints applied to the viscoelastic model presented, it is chosen to define a 1D system of wave propagation. In that case, unit vectors defined in eqn (8) are identical such that $p = \kappa = x$ where x denotes the 1D direction.

3. SYSTEM OF WAVE PROPAGATION

Let us consider the plane wave propagation along the x axis as illustrated in Fig. 1. This general 1D-system models an acoustic wall of thickness l excited at normal incidence by two harmonic sources located at $x = -h$ and at $x = d$. The first source is taken as the reference. Therefore, the propagation system is defined using three media: the forward medium (f), the studied medium (s) and the transmitted medium (t) related to subscripts f, s and t, respectively.

It is assumed that each of these three media (f), (s) and (t) is described by using the harmonic viscoelastic model given by eqns (5)–(6). Our aim is to analyse the energy dissipation through the medium (s) during the harmonic plane wave propagation. Hence, both media (f) and (t) are assumed well known.

Let $A_{s,f}$ or$_t$ be the amplitudes of the waves propagating along the increasing x and $B_{s,f}$ or$_t$ the amplitudes of the waves propagating in the opposite direction. These wave amplitudes are complex quantities. Since reflected waves are taken into account, the harmonic displacement $u_s(x)$ related to the medium (s) along the x axis is written as follows:

$$u_s(x) = A_s \exp[-jk_s^* x] - B_s \exp[+jk_s^* x],$$

$$\text{then} \quad \varepsilon_s(x) = \frac{\partial u_s}{\partial x} = -jk_s^*(A_s \exp[-jk_s^* x] + B_s \exp[jk_s^* x]). \tag{11}$$

The $\exp[j\omega t]$ notation is omitted for sake of conciseness. Using the constitutive law given by eqn (4), the solution of the equation of motion eqn (3) is given by eqn (11). Therefore, the wave number $k_s^*(\omega)$ is defined such that:

$$k_s^*(\omega) = \omega \sqrt{\frac{\rho_s^*(\omega)}{G_s^*(\omega)}} = \alpha(\omega) - j\gamma(\omega) \tag{12}$$

where $\alpha(\omega) \geqslant 0$ (m^{-1}) is related to the phase velocity, and $\gamma(\omega) \geqslant 0$ (m^{-1}) is the decay of the wave.

In addition to the wave number, the characteristic impedance $z^*(\omega)$ defined by eqn (10) gives a comprehensive description of the dynamic behaviour:

$$\omega z_s^*(\omega) = k_s^*(\omega) G_s^*(\omega) \quad \text{then}\, z_s^*(\omega) = \sqrt{\rho_s^*(\omega) G_s^*(\omega)} = z_s^r(\omega) - jz_s^i(\omega). \tag{13}$$

The boundary conditions express the continuity of velocities and stresses on both sides of the studied medium (s):

$$\left.\begin{aligned} \sigma_f(0) &= \sigma_s(0) \\ v_f(0) &= v_s(0) \end{aligned}\right\} \quad x = 0, \tag{14}$$

$$\left.\begin{aligned} \sigma_s(l) &= \sigma_t(l) \\ v_s(l) &= v_t(l) \end{aligned}\right\} \quad x = l. \tag{15}$$

The constitutive functions and the wave propagation system have been presented. To define the intrinsic dissipation related to medium (s), it is proposed to give the energy balance related to the wave propagation.

4. ENERGY BALANCE

The energy dissipated through medium (s) is that part of the incident energy which is neither reflected nor transmitted. The incoming energy impinging the studied wall at $x = 0$ is defined using the incoming power provided from the source located at $x = -h$. For the present 1D-problem, real parts (noted $\mathscr{R}(\,)$) of complex powers are active powers in watt/m^2 and designate energy flux intensities (Caviglia and Morro, 1992). Hence, the chosen incident intensity $\mathscr{I}_{\text{inc}}$ is (bar over variables denotes complex conjugate):

$$\mathscr{I}_{\text{inc}} = \mathscr{R}(\tfrac{1}{2}[-\sigma_f(0)]\bar{v}_f(0)) = \tfrac{1}{2}\omega^2 z_f^r |A_f|^2. \tag{16}$$

Towards this incident energy, the energy balance applied to medium (s) is written with adimensional energy coefficients lying between 0 and 1. The reflection coefficient **R** gives the part of the reflected energy propagating in the medium (f). The transmission coefficient **T** gives the transmitted energy propagating in the medium (t) and does not include the energy from the second source located at $x = d$. Therefore, the total intrinsic dissipation coefficient **D** is given by the following acoustic energy balance:

$$\mathbf{D} = 1 - \mathbf{R} - \mathbf{T}. \tag{17}$$

Following the principle of the virtual work rate, the energy balance related to the bounded studied medium (s) is obtained in terms of complex powers $\mathscr{P}_{\text{ext}}^*(\omega)$, $\mathscr{P}_{\text{int}}^*(\omega)$ and

$\mathscr{P}^*_{\rm acc}(\omega)$ related to the internal forces, to the external forces and to the inertial forces, respectively (Gorog, 1995). These quantities are defined on a time-period and therefore, the energy balance is written as follows:

$$\mathscr{P}^*_{\rm ext}(\omega)+\mathscr{P}^*_{\rm int}(\omega) = \mathscr{P}^*_{\rm acc}(\omega) \tag{18}$$

with

$$\mathscr{P}^*_{\rm ext}(\omega) = \tfrac{1}{2}(-\sigma_{\rm s}(0))\bar{v}_{\rm s}(0)-\tfrac{1}{2}(-\sigma_{\rm s}(l)\bar{v}_{\rm s}(l), \tag{19}$$

$$\mathscr{P}^*_{\rm int}(\omega) = -\int_0^l \frac{1}{2}\sigma_{\rm s}(x)\overline{\dot{\varepsilon}_{\rm s}}(x)\,{\rm d}x, \tag{20}$$

$$\mathscr{P}^*_{\rm acc}(\omega) = \int_0^l \frac{1}{2}\rho^*_{\rm s}(\omega)\dot{v}_{\rm s}(x)\bar{v}_{\rm s}(x)\,{\rm d}x. \tag{21}$$

In eqn (18), real parts are identified as dissipated energies while imaginary parts are interpreted as exchanged energies. The first and the second principle of continuum thermodynamics (Maugin, 1992) allow to exhibit an internal dissipation term in relation to the strain power. According to this result, an internal dissipation related to the bulk internal response during the wave propagation through the medium (s) is defined using a non-dimensional dissipative coefficient **E** such that:

$$\mathbf{E} = -\frac{\mathscr{R}(\mathscr{P}^*_{\rm int})}{\mathscr{I}_{\rm inc}}. \tag{22}$$

The method presented consists in calculating the coefficient **E** using the energy balance eqn (18) with eqns (19), (20) and (21) and the boundary conditions eqns (14) and (15).

5. DYNAMIC DISSIPATION

Using the definition of the total intrinsic dissipation given by eqn (17), the dissipative coefficient **E** (eqn (22)) is obtained as follows:

$$\mathbf{E} = 1-\mathbf{R}-\mathbf{T}-\mathbf{\Delta} = 2\frac{G^{\rm i}_{\rm s}|k^*_{\rm s}|^2}{\omega z^r_f}\frac{|z^*_f|^2}{|D_f|^2}\chi^{(-)}, \tag{23}$$

$$\mathbf{\Delta} = 2\omega\rho^{\rm i}_{\rm s}\frac{1}{z^r_f}\frac{|z^*_f|^2}{|D_f|^2}\chi^{(+)},$$

$$\mathbf{R} = R_f-2\frac{z^{\rm i}_f}{z^r_f}\mathscr{F}(r_f),$$

$$\mathbf{T} = T\frac{z^r_t}{z^r_f}\left\{1-\left(R_t-2\frac{z^i_t}{z^r_t}\mathscr{F}(r_t)\right)\right\}. \tag{24}$$

where

$$\chi^{(-)} = \frac{-1}{\gamma}U+\frac{1}{\alpha}V,\quad U = |\eta|^2\exp[-4\gamma l]-1+(1-|\eta|^2)\exp[-2\gamma l],$$

$$\chi^{(+)} = \frac{1}{\gamma}U+\frac{1}{\alpha}V,\quad V = 2\exp[-2\gamma l]\{\eta^i(1-\cos 2\alpha l)+\eta^r\sin 2\alpha l\},$$

$$\frac{B_{\rm s}}{A_{\rm s}} = \eta\exp[-2jk^*_{\rm s}l],\quad \eta = \frac{N_\eta}{D_\eta} = \frac{r_t(z^*_t+z^*_{\rm s})+(z^*_t-z^*_{\rm s})}{r_t(z^*_t-z^*_{\rm s})+(z^*_t+z^*_{\rm s})} = \eta^r+j\eta^{\rm i},$$

$$r_t = \frac{B_t}{A_t}, \quad r_f = \frac{B_f}{A_f}, \quad r_f = \frac{N_f}{D_f} = \frac{(z_s^* - z_f^*) + \frac{B_s}{A_s}(z_s^* + z_f^*)}{(z_s^* + z_f^*) + \frac{B_s}{A_s}(z_s^* - z_f^*)},$$

$$t = \frac{A_t}{A_f} = \exp[-jk_s^* l]\frac{4z_s^* z_f^*}{d_f D_\eta}, \quad R_f = |r_f|^2, \quad T = |t|^2, \quad R_t = |r_t|^2.$$

$\mathscr{F}()$ denotes imaginary part and the ω dependence has been omitted for conciseness. Quantities $N_{f\,\mathrm{or}\,\eta}$ and $D_{f\,\mathrm{or}\,\eta}$ denote numerator and denominator of fractions defining r_f or η, respectively. The present results are given for any boundary condition defined by η. The frequency dependent functions $\chi^{(-)}$ and $\chi^{(+)}$ can be interpreted as particular lengths associated with the wave propagation.

Two acoustic boundary conditions are usually used. Firstly, the studied medium is rigidly backed by an impervious wall. In that case, there is no transmission and a rigid termination (RT) occurs at $x = l$ such that:

$$r_t = 0, \quad z_t^* \to \infty, \quad T = 0 \quad \text{then } \eta = 1. \tag{25}$$

Secondly, there is no source located at $x \geqslant l$. Therefore the transmission tends to infinity and the medium (s) is defined as a free material (FM):

$$\exists A_t \quad \text{and} \quad B_t = 0 \Rightarrow r_t = 0 \quad \text{then } \eta = \frac{z_t^* - z_s^*}{z_t^* + z_s^*}. \tag{26}$$

The presented result eqn (23) shows that the total intrinsic dissipation **D** defined by eqn (17) comes from two different kinds of viscous dissipative effects according to the two dissipation coefficients **E** and **Δ** such that:

$$\mathbf{D} = \mathbf{E} + \mathbf{\Delta}. \tag{27}$$

The definition given by eqn (17) results from the energy balance related to the system of wave propagation (acoustic description) while the definition given by eqn (27) results from the evaluation of the internal dissipation related to the energy balance applied to the studied medium (s) (mechanical description).

Using eqns (21) and (24), coefficient **Δ** is defined as follows

$$\mathbf{\Delta} = \frac{\mathscr{R}(\mathscr{P}_{\mathrm{acc}}^*)}{\mathscr{I}_{\mathrm{inc}}}, \tag{28}$$

and, according to eqns (18) and (17), the energy coefficient **D** becomes hence:

$$\mathbf{D} = \frac{\mathscr{R}(\mathscr{P}_{\mathrm{ext}}^*)}{\mathscr{I}_{\mathrm{inc}}}. \tag{29}$$

The first kind of dissipative effect is related to the viscous bulk since the coefficient **E** eqn (23) is directly commensurate to the imaginary part of the viscous modulus G_s^* (eqn (5)). The second dissipative coefficient **Δ** defined by eqn (28) shows the importance of the viscous mass effects in the dynamic description of the medium. This coefficient (eqn (24)) is directly commensurate to the imaginary part of the complex mass function ρ_s^* (eqn (6)).

The direct calculation of eqns (19)–(21) gives:

$$\mathscr{P}_{\mathrm{ext}}^* = \tfrac{1}{2}|A_f|^2\omega^2\{z_f^*[1 - R_f - \bar{r}_f + r_f] - z_t^* T[1 - R_t - \bar{r}_t + r_t]\}$$

or

$$\mathscr{P}^*_{\rm ext} = -\tfrac{1}{2}|A|^2\omega^2 z^*_{\rm s}\chi^* \quad \text{with}\ \chi^* = U + jV,$$

and

$$\mathscr{P}^*_{\rm int} = -\tfrac{1}{4}|A|^2\omega^2 z^*_{\rm s}\overline{(jk^*_{\rm s})}\chi^{(-)}, \quad \mathscr{P}^*_{\rm acc} = -\tfrac{1}{4}|A|^2\omega^2 j\omega\rho^*_{\rm s}\chi^{(+)},$$

and permits to verify eqn (18).

Different coefficients have been defined to calculate the intrinsic dissipation during the wave propagation. The problem is now to verify the laws of continuum thermodynamics to define a consistent model.

6. THERMODYNAMIC ANALYSIS

The second law of thermodynamics imposes that the total intrinsic dissipation **D** (eqn (27)) must be positive:

$$\mathbf{D} = \mathbf{E} + \mathbf{\Delta} \geqslant 0. \tag{30}$$

Since two distinct viscous dissipative effects have been defined with coefficients **E** (eqn (23)) and **Δ** (eqn (24)), this thermodynamic condition is applied to each of them. This means that each energy coefficient must be positive to verify eqn (refeq: positive **D**):

$$\mathbf{E} \geqslant 0 \quad \text{and} \quad \mathbf{\Delta} \geqslant 0. \tag{31}$$

Let us note that eqn (20) with eqn (4) on the one hand and eqn (21) on the other hand can be rewritten:

$$\mathscr{P}^*_{\rm int}(\omega) = -\frac{1}{2}j\omega G^*_{\rm s}(\omega)\int_0^l |\varepsilon_{\rm s}(x)|^2\,{\rm d}x \quad \text{and} \quad \mathscr{P}^*_{\rm acc}(\omega) = \frac{1}{2}j\omega\rho^*_{\rm s}(\omega)\int_0^l |v_{\rm s}(x)|^2\,{\rm d}x.$$

Therefore, using the definitions eqn (22) and (28), respectively, conditions are found to verify the two thermodynamic constraints of eqn (31). First, coefficient **E** (eqn (23)) is positive if and only if the imaginary part of the viscous modulus is positive. Second, coefficient **Δ** (eqn (24)) is positive if and only if the imaginary part of the complex density is negative.

Using these results, it is obtained that the thermodynamic constraints impose the following conditions to verify eqn (30) and eqn (31):

$$\forall\omega, \quad G^r_{\rm s} \geqslant 0, \quad G^i_{\rm s} \geqslant 0 \quad \text{and} \quad \rho^r_{\rm s} \geqslant 0, \quad \rho^i_{\rm s} \leqslant 0, \forall\eta, \quad U < 0, \quad \frac{\alpha}{\gamma}|U| \leqslant V \leqslant \frac{\alpha}{\gamma}|U|,$$

$$\chi^{(-)} \geqslant 0, \quad \chi^{(+)} \leqslant 0, \quad \frac{z^r_{\rm s}}{z^i_{\rm s}} \geqslant \frac{\alpha}{\gamma} \geqslant 0. \tag{32}$$

These conditions have to be verified for any boundary conditions and for all angular frequency. They are the thermodynamic restrictions needed to define a comprehensive harmonic viscoelastic model. It is worth emphasising that it shows that the imaginary part of the complex density function eqn (6) results from the correct modelling of the viscous body force as recovering force at the descriptive scale of the continuum.

In a general thermodynamic time domain description, dissipative behaviour is explained with internal variables β which express the irreversible response of the medium

(Maugin, 1992). Hence, for a viscoelastic medium of volume Ω, the intrinsic dissipation is written as follows:

$$\mathscr{D} = \int_{\Omega} \sigma^{\text{irr}} : \dot{\varepsilon}\, d\Omega + \int_{\Omega} \Upsilon \cdot \dot{\beta}\, d\Omega \geqslant 0.$$

where σ^{irr} denotes the irreversible stress tensor and Υ the thermodynamic force associated with internal variables β.

For the presented frequency analysis, the first term corresponds to the phase delay between stress and strain because of the viscous modulus. It is a viscous dissipation depending on the bulk rigidity. Therefore, this term is related to the energy coefficient $\mathbf{E}$. The second term depends on internal variables β. As a result, it is connected to the dynamic term $\mathbf{\Delta}$. At the scale under dissipation, this means that the dissipative inertia terms can be modelled using internal hidden variables.

Hence, it remains to describe the evolution laws of these variables and also to define the thermodynamic dissipative energy function. The main difficulty consists in distinguishing the different terms of the dissipated energy function in the time domain. In fact, Caviglia and Morro (1992) indicate that there exist inequivalent choices of energy density and flux for viscoelastic media. Using a model with internal variables (Maugin, 1992), the problem consists in defining the evolution laws of the internal variables associated with an internal dissipation potential. This potential is used to formulate the thermodynamic dissipation energy function.

7. EXAMPLE

Some sound absorbing materials such as slight glass wools or foams are described as open porous media with rigid skeleton. From the acoustic point of view and using a harmonic phenomenological approach, these absorbing porous media are modelled as homogeneous viscoelastic equivalent fluids at the macroscopic scale. This model describes the dynamic behaviour in the frequency domain according to both harmonic functions eqns (5) and (6). These functions are defined using microscopic properties related to the porous network and to the ambient saturating air. As an example, we have chosen the Allard's (1993) model in studying a polyurethane foam of thickness $l = 0.1$ m for which the experimental results agree with the porous model.

Therefore, the studied medium is a viscoelastic continuum characterised by:

$$\rho_s^*(\omega) = \frac{1}{\phi}\tilde{\alpha}(\omega)\rho_0 = \frac{\alpha}{\phi}\rho_0\left[1 + \frac{\phi\sigma}{j\omega\rho_0\alpha}G_J(\omega)\right],$$

$$G_s^*(\omega) = \frac{1}{\phi}\gamma P_0\left\{\gamma - (\gamma-1)\left[1 + \frac{\sigma'\phi}{j\omega B^2\rho_0\alpha}G'_J(B^2\omega)\right]^{-1}\right\}^{-1},$$

with

$$G_J(\omega) = \left(1 + \frac{4j\omega\rho_0\alpha^2\eta}{\sigma^2\Lambda^2\phi^2}\right)^{1/2}, \quad \Lambda = \frac{1}{c}\left(\frac{8\eta\alpha}{\sigma\phi}\right)^{1/2},$$

$$G'_J(B^2\omega) = \left(1 + \frac{4j\omega B^2\rho_0\alpha^2\eta}{\sigma^2\Lambda^2\phi^2}\right)^{1/2}, \quad \Lambda' = \left(\frac{8\eta\alpha}{\sigma'\phi}\right)^{1/2}.$$

The function G_J of positive real part describes the viscous air flow through the porous network. Λ, σ and Λ', σ' are characteristic parameters related to viscous and thermal effects at the scale of the pores. c, ϕ and α are geometrical parameters of the porous network. η,

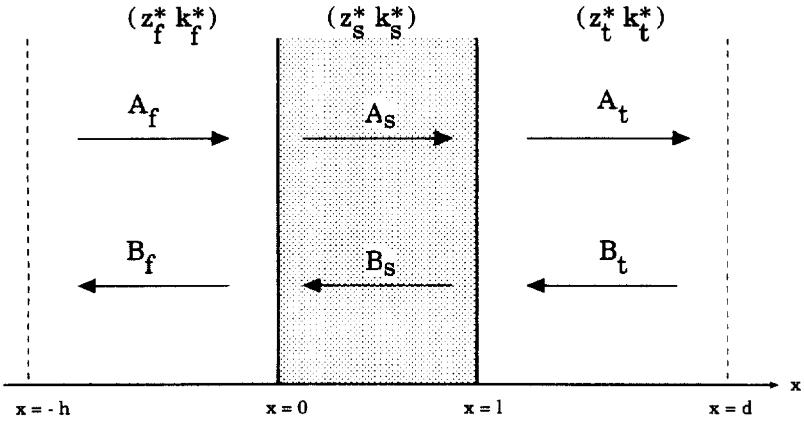

Fig. 1. System of wave propagation.

ρ_0, γ, B^2 and P_0 are characteristic coefficients of the saturating air. ρ_s is the density of the solid part. These parameters take the following values:

$$\eta = 1.84\times10^{-5}\,\mathrm{kg\,m^{-1}\,s^{-1}},\quad \gamma = 1.4,\quad B^2 = 0.71,\quad P_0 = 1.0132\times10^5\,\mathrm{Pa},$$

$$\rho_0 = 1.3\,\mathrm{kg/m^3},\quad \phi = 1,\quad \sigma = 9000\,\mathrm{kg\,m^{-2}\,s^{-1}},\quad \Lambda = 1.92\times10^{-4}\,\mathrm{m},$$

$$\rho_s = 16\,\mathrm{kg/m^3},\quad \alpha = 1,\quad \sigma' = 998.26\,\mathrm{kg\,m^{-2}\,s^{-1}},\quad \Lambda' = 3.84\times10^{-4}\,\mathrm{m}.$$

It is verified that the descriptive functions are consistent with the thermodynamic conditions of eqn (32). The density function models the viscous mass motion related to the fluid flow while the bulk function takes into account the thermal exchange between the two phases. At the scale of the continuum, these dissipative effects appear as viscous damping. The two dissipative coefficients **E** (eqn (23)) and **Δ** (eqn (24)) are evaluated for both conditions eqn (25) (Rigid termination: Fig. 2) and eqn (26) (Free material: Fig. 3) for the frequency range from 0 to 10,000 Hz. Both Figs 2 and 3 show that the main dissipativity arises from the viscous mass effects (**Δ**). The model shows that these effects are due to the air flow conduction. Therefore, the most important viscous dissipation comes from the viscous air flow through the porous network. This physical phenomenon is specifically

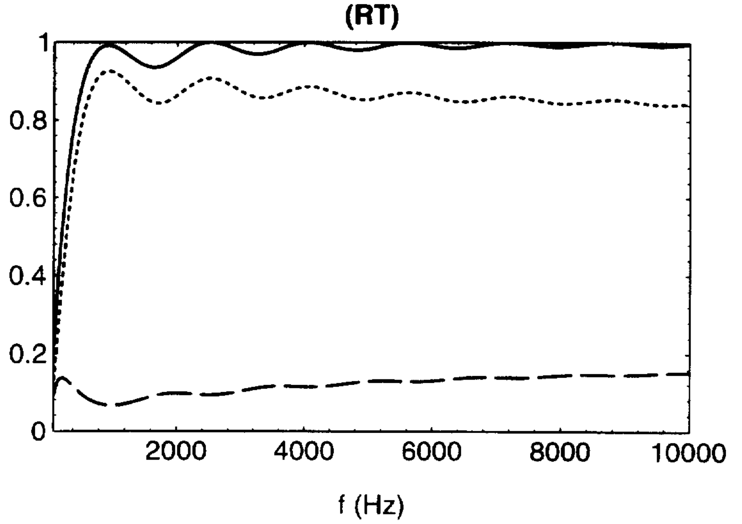

Fig. 2. Intrinsic dissipation coefficients: **E** (---), **Δ** (...), **D** (—). Rigid boundary.

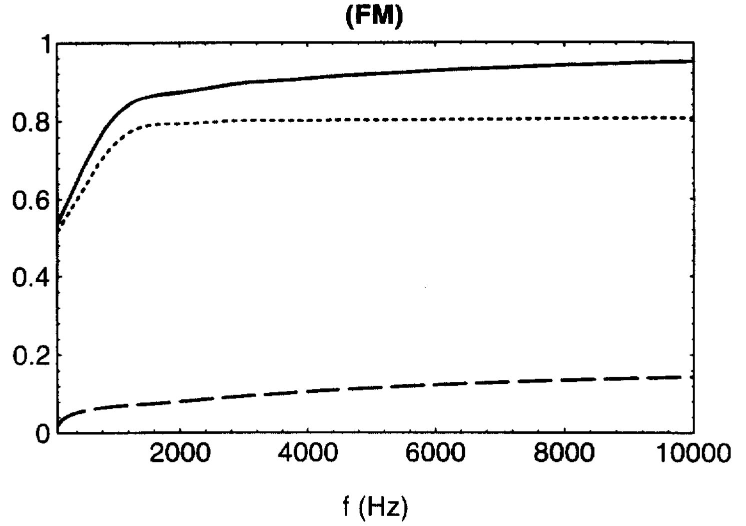

Fig. 3. Intrinsic dissipation coefficients: **E** (- - -), **Δ** (. . .), **D** (—). Free boundary.

described in the complex density formulation and is understood here as restoring body force.

8. CONCLUSIONS

This paper presented the thermodynamic restrictions necessary to develop comprehensive models to study the harmonic plane wave propagation through absorbing viscoelastic media. The dynamic behaviour of the studied medium is described using two distinct complex frequency dependent functions related to the viscous bulk rigidity and to viscous body force, respectively. The frequency dependent complex bulk function results from the spectral transform of the viscoelastic constitutive law. It has been shown that the frequency dependent complex density accounts for the spectral transform of viscous body force. Such harmonic viscoelastic models are used to describe the frequency behaviour of sound absorbing materials. These materials are considered as homogeneous media at the macroscopic scale.

A simple one-dimensional wave propagation system has been proposed to model an acoustic wall excited by normal harmonic plane waves. The method presented consists in calculating the dynamic intrinsic dissipation during the wave propagation. Within the framework of the thermodynamics of irreversible processes, it has been shown that two dissipations of different origins have to be distinguished. These two different viscous origins have been explained using the two kinds of descriptive functions related to the bulk response and to inertia effects. The classical continuum thermodynamics has been used in the harmonic context to propose a correct definition of the dynamic intrinsic dissipation. The example of the model of sound absorbing materials described as rigid open porous media has been shown that the dominant dissipative mechanism can be those modelled through the density function.

The thermodynamic analysis proposed allows one to formulate a consistent model of the dissipative phenomena defined by a viscous rigidity and a viscous mass motion. Especially, this means that the dissipative mechanisms described through the complex density function must be understood as recovering body forces. These forces introduce a dynamic viscosity at the descriptive scale of the continuum but they provide from a physical phenomenon at the microscopic scale as, for instance, a fluid flow. The main difficulty consists in defining the model with internal variables to give a dissipation energy function using dissipation potential.

This paper attempted to analyse harmonic models from a thermodynamic point of view. Extension to a three-dimensional description requires consideration of the vectorial nature of the equation of motion. In that case, body forces are described by three scalar functions and, hence, it is suggested that several dissipative phenomena can be considered according to the different directions. It is worth mentioning that a vectorial definition of the characteristic impedance has been proposed. The total intrinsic dissipation will result from the sum of the two kinds of dissipation related to viscous rigidity and mass effects. Nevertheless, it appears to be not possible to conclude about each direction. The application of the thermodynamic restrictions can then lead to different conclusions about the sign of the descriptive variables. Moreover, the problem of the coupling between the propagating waves remains to be solved.

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